International Journal of Theoretical Physics, Vol. 33, No. 11, 1994

Nonstatic Gödel-Type Cosmological Model with Perfect Fluid and Heat Flow

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Received February 14, 1994

We obtain some cosmological models that are exact solutions of Einstein field equations. The metric utilized is the nonstatic Gödel-type cosmological model and the curvature source is a perfect fluid with heat flow. The solutions reported here are nonstatic generalizations of Gödel's rotating cosmos.

1. INTRODUCTION

In 1949 K. Gödel (Gödel, 1949) published the first cosmological model generated by a solution of the modified Einstein equations in which a cosmological repulsive term (Λg_{ik}) has been added. The congruence of the geodesics of this model ($u^i = \delta^i_0$) has no shear, no expansion, and no acceleration, but presents a constant rotation of matter relative to the compass of inertia. After this discovery, many attempts were made to construct more general solutions which take the expansion and/or shear into account besides rotation.

Since that time, there has been considerable interest in the general problem of rotating cosmological models (see, for example, Bradley and Sviestins, 1984) as well as in the particular problem of finding new exact cosmological models of Gödel type. Dunn (1989) found two-fluid solutions to the field equations for Gödel-type spacetimes. Rotating cosmological models with viscous fluid and heat flux have been studied by Koppar and Patel (1988*a,b*). Rebouças and Tiomno (1985) found a class of inhomogeneous Gödel-type models. These models have a scalar field plus a fluid with heat flux as the source of geometries.

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The main purpose of the present work is to derive some nonstatic rotating cosmological models with perfect fluid and heat flow.

2. EINSTEIN'S FIELD EQUATIONS

We consider a nonstatic Gödel-type metric of the form (Koppar and Patel, 1988*a*)

$$ds^2 = (dt + He^x\,dy)^2 - \frac{1}{2}H^2e^{2x}\,dy^2 - dx^2 - dz^2 \tag{1}$$

where H is a function of t alone.

The Einstein field equations for a perfect fluid with heat flow are

$$G_{ik}: R_{ik} - \frac{1}{2}Rg_{ik} + \Lambda g_{ik} = (\rho + p)u_iu_k - pg_{ik} + q_iu_k + q_ku_i \tag{2}$$

where u_i, ρ, and p are the four-velocity of matter, the energy density, and the pressure, respectively; q_i is the heat flow vector and satisfies the equation $q_iu^i = 0$. We use geometrized units so that $8\pi G = c = 1$.

In the comoving coordinate system $[u^i = (1, 0, 0, 0),\ u_i = (1, 0, He^x, 0)]$, the Einstein field equations ($i, k = 0, 1, 2, 3$; $x^0 = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$) are

$$G_{00}: \frac{1}{2} + \Lambda = \rho + 2q_0 \tag{3}$$

$$G_{01} \equiv G_{12}: q_1 = \frac{\dot{H}}{H} \tag{4}$$

$$G_{02}: \frac{1}{2} + \Lambda = \rho + q_0 + H^{-1}e^{-x}q_2 \tag{5}$$

$$G_{03} \equiv G_{23}: q_3 = 0 \tag{6}$$

$$G_{11} \equiv G_{33}: \frac{\ddot{H}}{H} + \frac{1}{2} - \Lambda = p \tag{7}$$

$$G_{22}: \frac{3}{2} + \Lambda = 2\rho + p + 4H^{-1}e^{-x}q_2 \tag{8}$$

where the dot represent derivative with respect to time t. The remaining equation (G_{13}) is identically zero.

From equation (6) and the condition $q_iu^i = 0$, the heat flow vector is restricted to

$$q_i = (0, q_1, q_2, 0) \tag{9}$$

3. SOLUTIONS OF THE FIELD EQUATIONS

From equations (3) and (9), we obtain

$$\rho = \frac{1}{2} + \Lambda \tag{10}$$

From equations (5), (9), and (10), we have $q_2 = 0$. Therefore, the heat flow vector q_i is

$$q_i = (0, q, 0, 0) \tag{11}$$

where q ($= q_1$) is a function of time t to be determined from equation (4).

Using (8), (10), and (11), we obtain

$$p = \frac{1}{2} - \Lambda \tag{12}$$

From equations (7) and (12), we have $\ddot{H} = 0$, i.e.,

$$H = at + b \tag{13}$$

where a and b are constants.

If we substitute equation (13) in (4), we obtain

$$q = \frac{\dot{H}}{H} = \frac{a}{at + b} \tag{14}$$

4. KINEMATIC QUANTITIES

The expansion $\Theta = u^i{}_{;i}$ given by

$$\Theta = \frac{\dot{H}}{H} = \frac{a}{at + b} \tag{15}$$

The shear scalar (σ^2) and the rotation (Ω^2) of the four-velocity vector $u_i = (1, 0, He^x, 0)$ are determined as

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{H}}{H}\right)^2 = \frac{1}{3}\left(\frac{a}{at + b}\right)^2, \qquad \Omega^2 = \frac{1}{2} \tag{16}$$

Thus the vorticity remains constant along the whole history of our universe. The acceleration vector $\dot{u}_i$ is given by

$$\dot{u}_i = u^i{}_{;k} u^k = (0, 0, \dot{H}e^x, 0) = (0, 0, ae^x, 0) \tag{17}$$

It is not difficult to see that the flux q_i turns out to be in the direction orthogonal to the plane generated by the vorticity and the acceleration. As $\dot{u}_i \neq 0$ ($a \neq 0$), the streamlines of the perfect fluid filling the universe are not geodetic.

It is easy to see that

$$\frac{\sigma}{\Theta}=\frac{1}{\sqrt{3}}\cong 0.577$$

for our models. The present upper limit of σ/Θ is 10^{-3} obtained from indirect arguments concerning the isotropy of the premordial blackbody radiation (Collins *et al.*, 1980). The ratio σ/Θ for our models is considerably greater than its present value. This fact indicates that our solutions represent the early stages of evolution of the universe.

From equations (10) and (12), it is clear that ρ and p are constants. From equations (14)–(16), it is easily seen that q, Θ, and σ^2 are functions of time t.

If we set $a=0$ in the above results, H becomes a constant and consequently our solution becomes the Gödel solution ($q=\Theta=\sigma^2=0$).

We can recognize the constant a/b as the value assumed by the expansion at the origin of time ($t=0, q=\Theta=a/b$). At the final stage of the evolution as $t\to\infty, q=\Theta=\sigma^2=0$.

The phenomenological expression for the heat conduction is given by

$$q_i=\kappa(T_{,k}+T\dot{u}_k)h_i^k, \qquad h_i^k=\delta_i^k-u^k u_i \tag{18}$$

where κ is the thermal conductivity and T is the temperature. Here it should be noted that the homogeneity consideration restricts the thermal conductivity κ to be a function of time t alone.

Equation (18) in view of (17) leads to

$$\kappa T_{,1}=q, \qquad T_{,2}+e^x(T\dot{H}-H\dot{T})=0 \tag{19}$$

Equations (19) are satisfied provided

$$\kappa=\frac{a}{\alpha(at+b)^2}, \qquad T=H(\alpha x+\beta)$$

where α and β are arbitrary constants. Thus, the thermal conductivity and the temperature are expressed in terms of the function H.

5. CONCLUSIONS

In the present paper, we have exhibited some exact cosmological solutions of Einstein's field equations which have expansion, rotation, and shear. The energy-momentum tensor is described by a perfect fluid with heat flow. The solutions are nonstatic generalizations of Gödel's stationary universe with perfect fluid and heat flow. When the time t is infinity, σ, q, and Θ become zero. Therefore, the ultimate fate of the above solution is Gödel's universe.

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